# LOCAL SYMMETRIES IN NETWORKS

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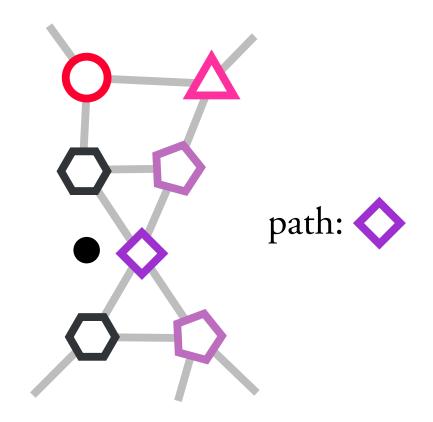


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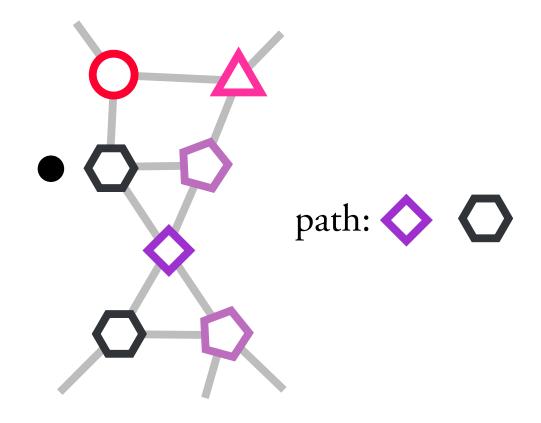


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- ★ What symmetries are relevant in networked systems.
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- ★ What does a center of local symmetry mean?

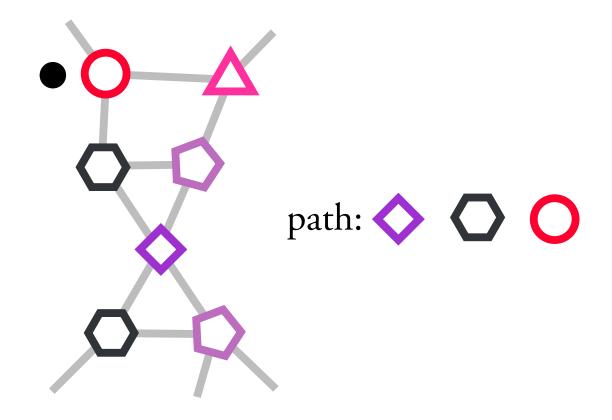




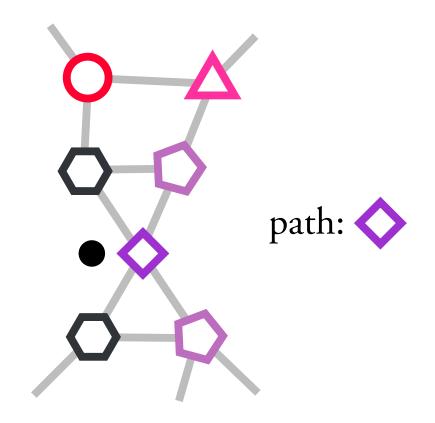




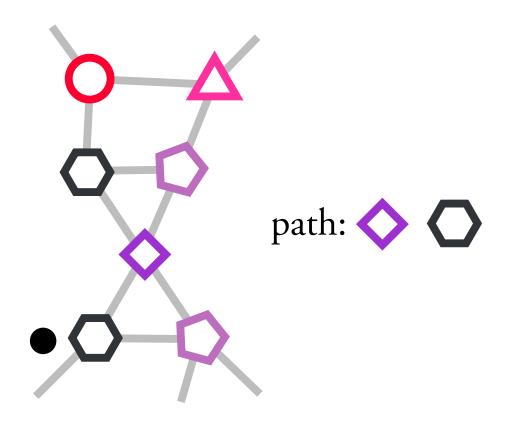




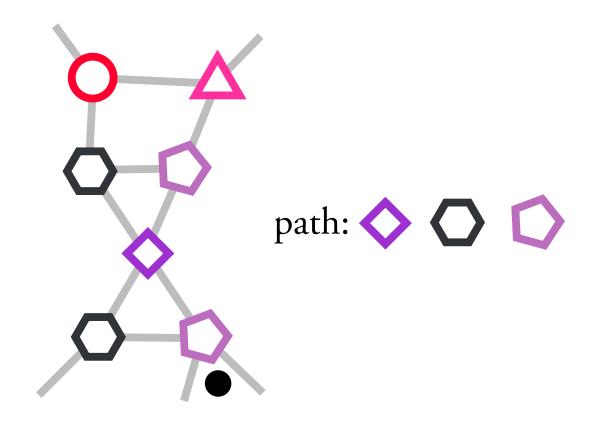














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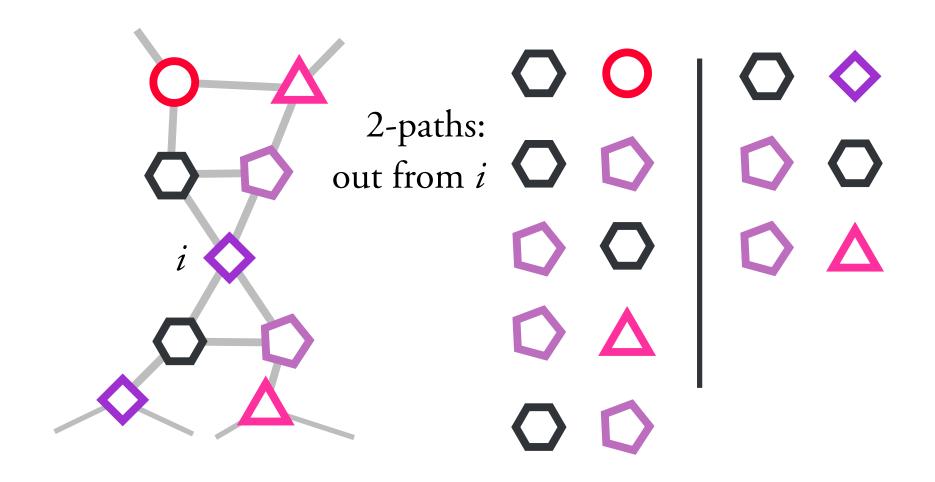


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- ★ If many paths out from a vertex are similar, that vertex is a center of symmetry.
- ★ The surrounding of the vertex is functionally invariant with respect to permutations of the paths.

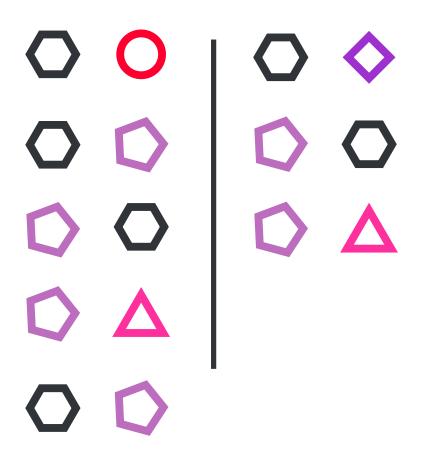






#### idea:

look at the pairs of vertices at a distance *d* of forked paths





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- count the fraction of overlapping such pairs



















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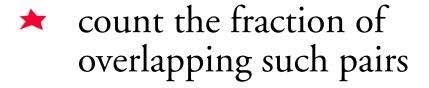




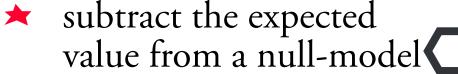
















#### idea:

- ★ look at the pairs of vertices at a distance d of forked paths
- count the fraction of overlapping such pairs
- subtract the expected value from a null-model

local symmetry coefficient



# functional similarity

What determines if a vertex is O or ?





How define equivalence classes?

**★** Degree.



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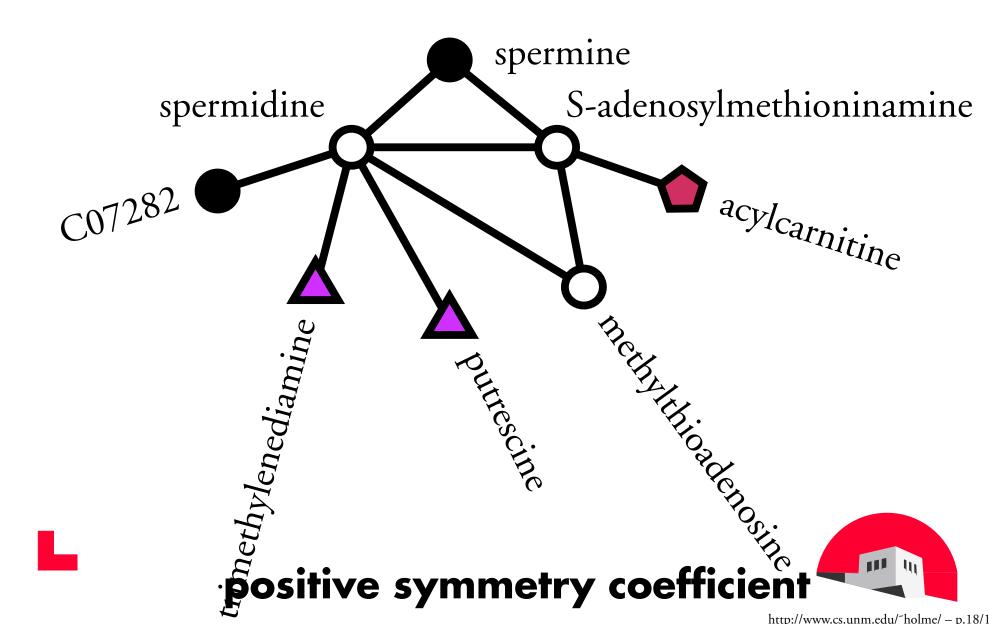


How define equivalence classes?

- ★ Degree.
- ★ Clustering coefficient (or any other vertex specific network measure).
- ★ External functionallity. E.g. protein function.



# degree sym.: human metabolism



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