Statistical physics approach to landscape modeling: Neighborhood interaction and aridity explain the forest distribution of the Southwest US

Etsuko Nonaka,[1] Petter Holme,[2] and Evangelos A. Coutsias[3]

1 Department of Biology, 2 Department of Computer Science, 3 Department of Mathematics and Statistics, University of New Mexico, Albuquerque, New Mexico 87131, USA.

1.Introduction

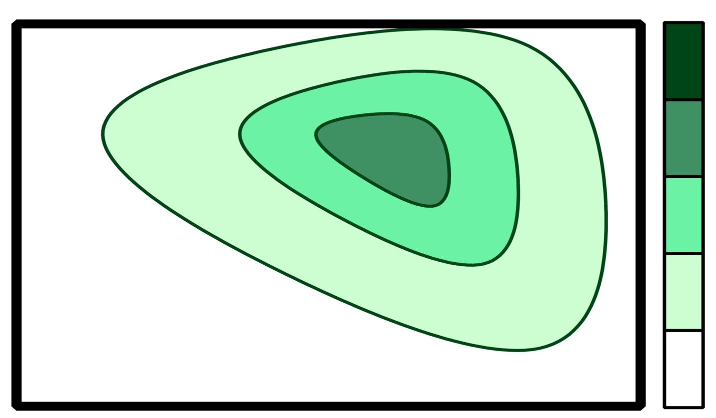
Patch boundaries or ecotones are fundamental components of heterogeneous landscapes. Both biotic and abiotic processes and their interactions are important mechanisms for their formation, and abrupt transitions can form along smooth environmental gradients (Yamamura 1976, DeAngelis et al. 1986, Wilson and Nisbet 1997). Such characteristics of boundaries suggest that they resemble phase transitions in physical systems: phase transitions are abrupt sudden changes in physical properties of a system with gradual changes in controlling parameter values. Phase transition is studied in statistical physics, a discipline that studies probabilistic behaviors of a large number of interacting particles. Borrowing the basic principles and tools of statistical physics, we investigated simple explanations for complex collective behavior of vegetation, forming landscape patterns at a large spatial scale.

2. Objective

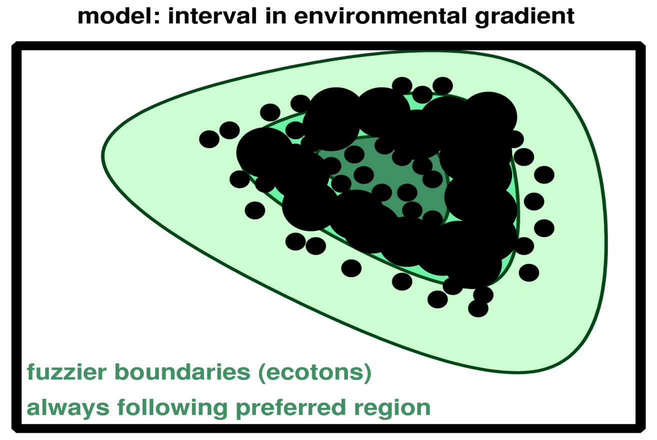
To understand the effects of neighborhood interactions and environmental gradients on formation of patch boundaries in landscapes using a simple simulation model of statistical physics

3. Idea

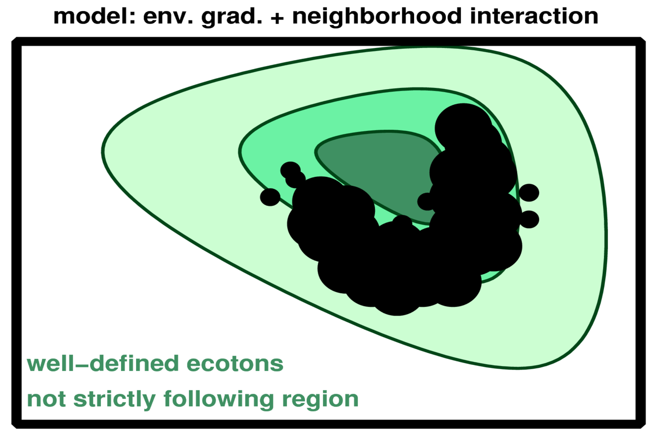
The landscape is a result of both the environmental situation and interaction between neighbors



model: interval in environmental gradient



model: env. grad. + neighborhood interaction

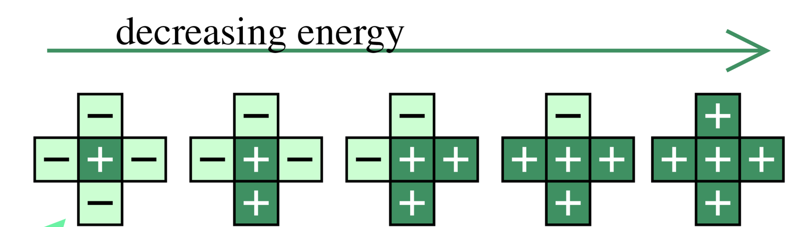


4. Methods

ii. Tool: The Ising model

We assume that the principal processes in landscape formation are **1.** coupling to environmental gradients **2.** neighbor interaction. The "maximum entropy principle" tells us that this is best described by the Ising model (familiar to physicists as a model for magnetism). The simplicity of the Ising model makes it very versatile. E.g. the canopy-gap size frequency distribution of tropical forests have been modeled (Katori et al. 1998, Kizaki and Katori 1999).

So let the landscape be described by a binary variable s_i: s_i = 1 is there is a certain forest type on pixel i, s_i = 0 otherwise



probability of $\{s_i\}$: $\frac{\exp(-E)}{Z}$ where

$$E = \sum_i \left[K \sum_j s_i s_j + h s_i \right]$$ and j is a neighbor of i

coupling to neighbors — coupling to environment

ii. Ecological interpretations of the parameters *K* and *h*

We further applied this model to large heterogeneous landscapes by varying the values of *K* and *h* over space. We interpreted in such a way that we can examine the hypothesis that nearest-neighbor interactions and environmental gradients are important factors for the formation of patch boundaries and general structure of landscapes.

- Nearest-neighbor interaction strength (*K*) → Nearest-neighbor positive interaction of a vegetation type, or spatial autocorrelation.
- External magnetic field (*h*) → Habitability along environmental gradients

iii. Modeled ecosystem

Pinyon-juniper (PJ) woodland in semi-arid landscapes of SW USA (Fig. 3)

- Forest type map (Zhu and Evans 1992) as the input data — Spatial resolution = 1km x 1km
- Environmental gradient: Budyko Aridity Index = PPT / PET
  - PPT = precipitation, PET = potential evapotranspiration
  - Gradient of water-limited and energy-limited environment
- Estimating the values of parameters *K* and *h* (Fig. 4)
  - Measured the probability of cell occupancy of PJ woodland given differing numbers of neighbors of the same type, using the forest type map
  - Then, *K* and *h* were estimated by the least square method, using the Boltzmann distribution law of entropy for the relationship between probability of occurrence and energy levels calculated by the Ising model.

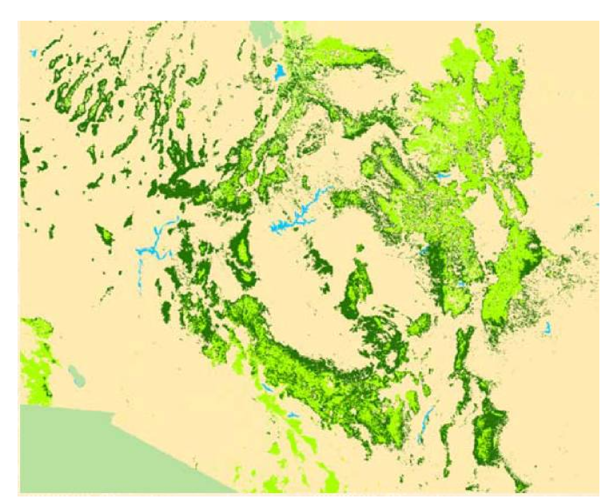


Figure 3. Pinyon-juniper woodland type (dark green) and other forest types (light green) in the SW USA. Subset landscapes were used in the simulations.

5. Results

- Can the Ising model recreate landscape structure solely based on nearest-neighbor interactions and an environmental gradient?
  - Yes, which suggests that these two mechanisms are important for pattern formation in landscapes (Fig. 5a,b). But there are some noticeable discrepancies.
- What are the effects of nearest-neighbor interaction and environmental gradients?
  - When *K* (neighborhood interaction) was turned off, abrupt boundaries did not form at the minimum energy level (Fig. 8).
  - When *h* (habitability gradient) was turned off, the locations of patches differed in every simulation (results not shown).

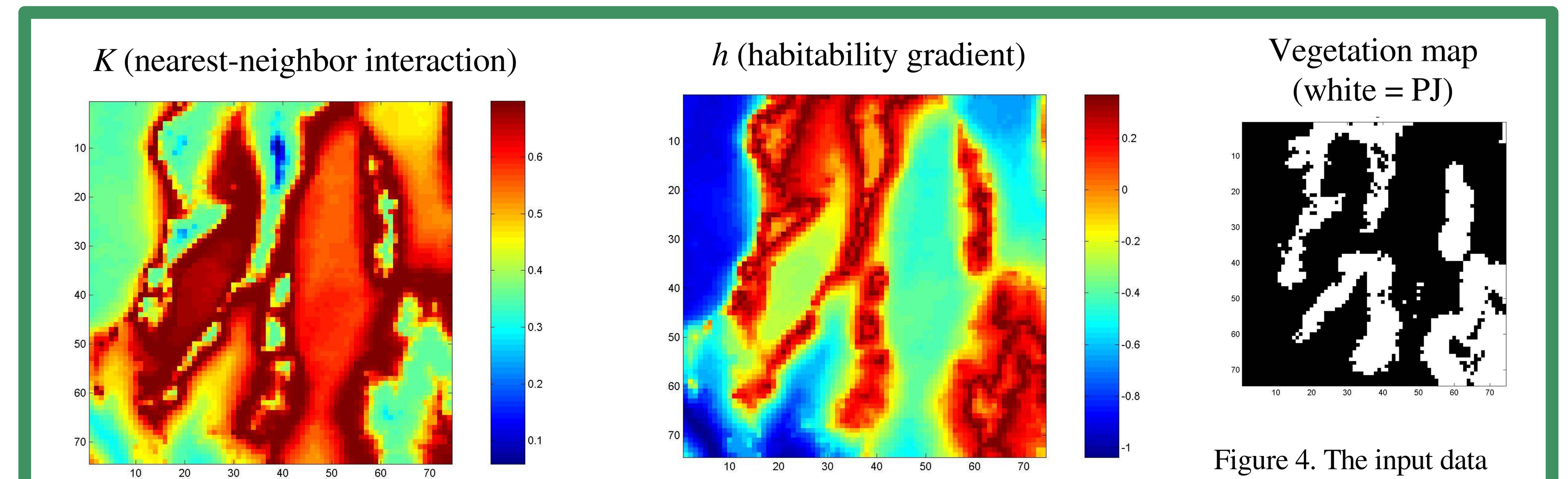


Figure 4. The input data

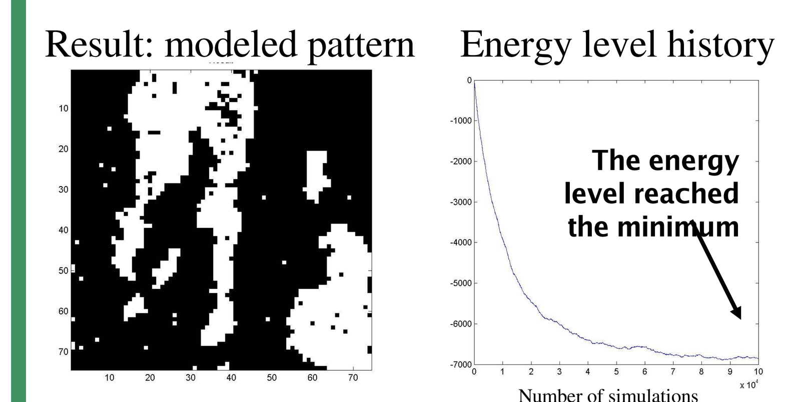


Figure 5a. An example result showing a typical pattern generated by the Ising model. The "energy" level of the landscape pattern was close to the minimum when the pattern resembled the real pattern.

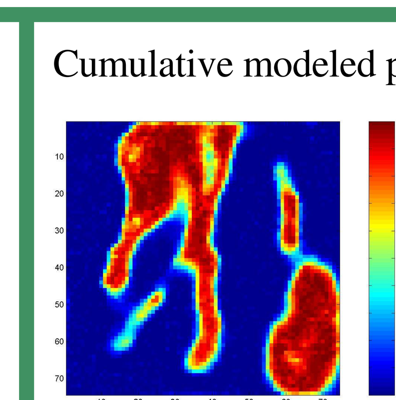


Figure 5b. Frequency of PJ occurrence at each cell location based on 100 simulations.

6. Ecological insights

- Abrupt patch boundaries or ecotones can be considered as spatial phase transitions in landscapes (Fig. 7).
- When there is no neighborhood interaction, patch boundaries are not as well defined. When there is no environmental gradient, locations of patches in space are not determined.
- Because the objective function of the Ising model is overall energy minimization, some process that is analogous to "energy" minimization may be a driving force of landscape pattern formation in some systems.

Figure 6. Scaling up to a larger part of the Southwest: Many features are recreated but regions of discrepancies exist.

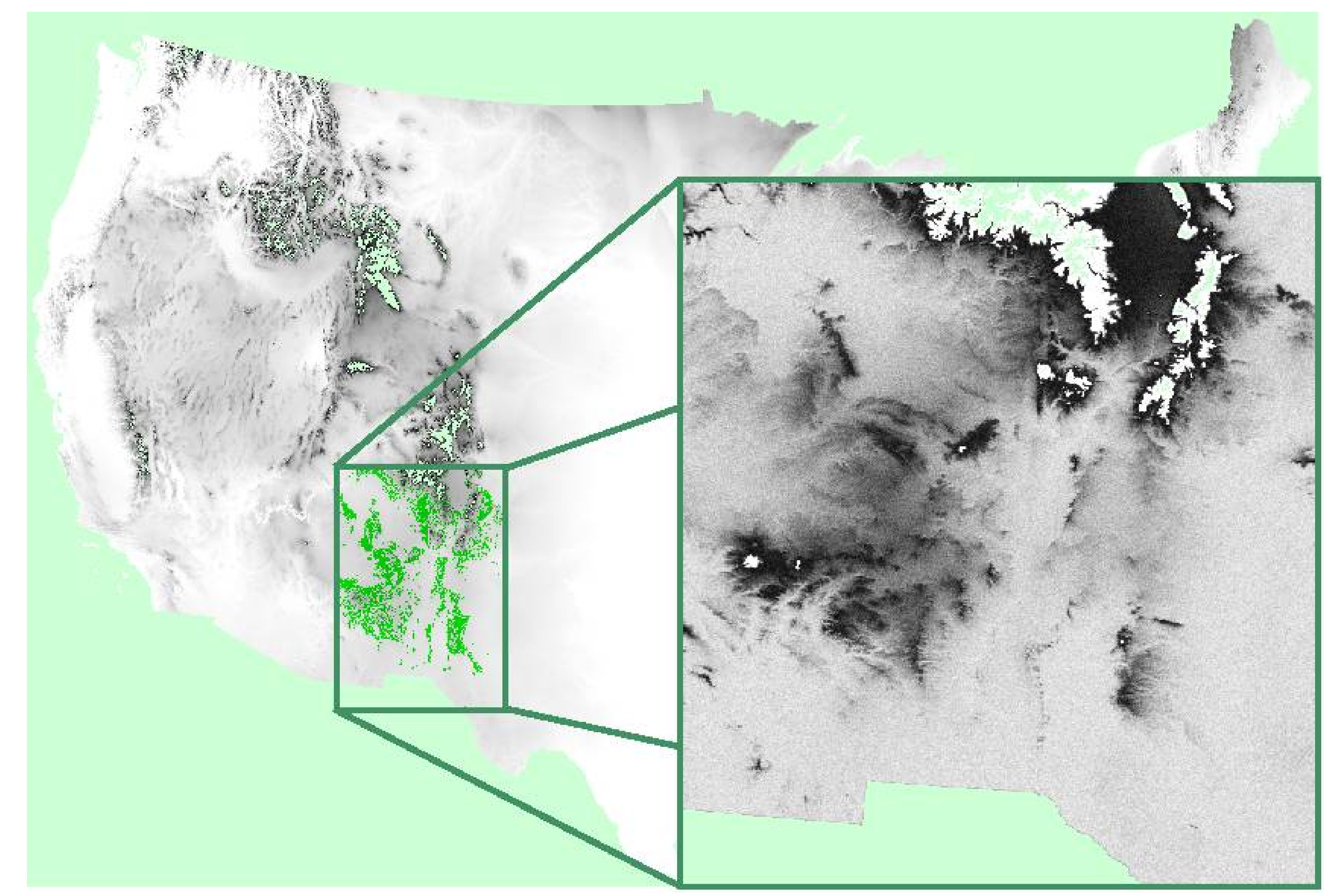
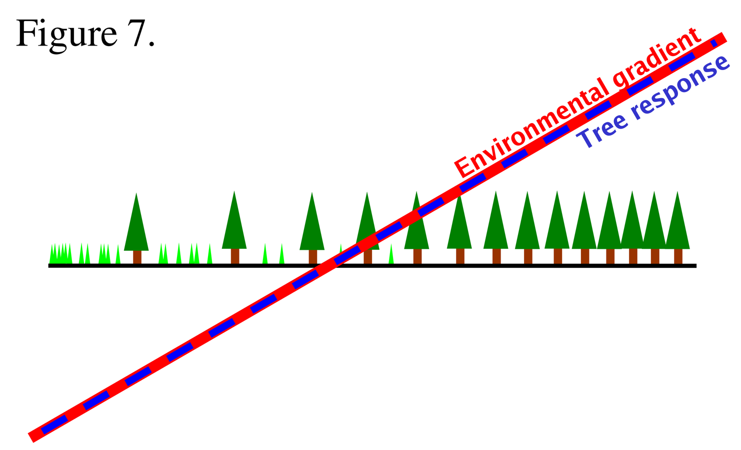
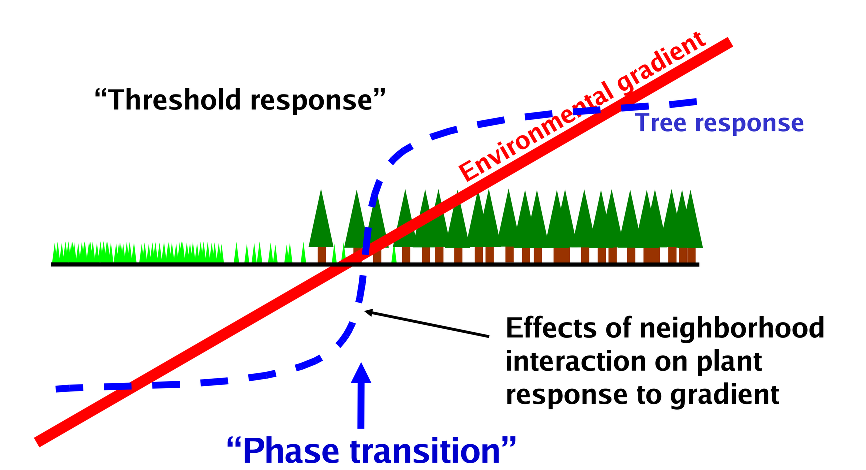


Figure 7.



← a) No neighborhood interaction (K = 0)—plant response to environmental gradients can be linear.

→ b) With neighborhood interaction (K > 0)—plant response to environmental gradients can be non-linear. A threshold response to gradual change in controlling parameter is called a "phase transition."



7. Limitations

- The real landscape pattern is assumed to be at the equilibrium. Historical contingency (e.g., chance dispersal, infrequent large disturbance, directionality of migration) is not incorporated.
- Only one forest-type is considered, coupling between forest-types is omitted.
- Response to environment within the PJ woodland type may change over a large space (i.e., ecotypes).