

# overload breakdown in evolving networks

Petter Holme  
Beom Jun Kim

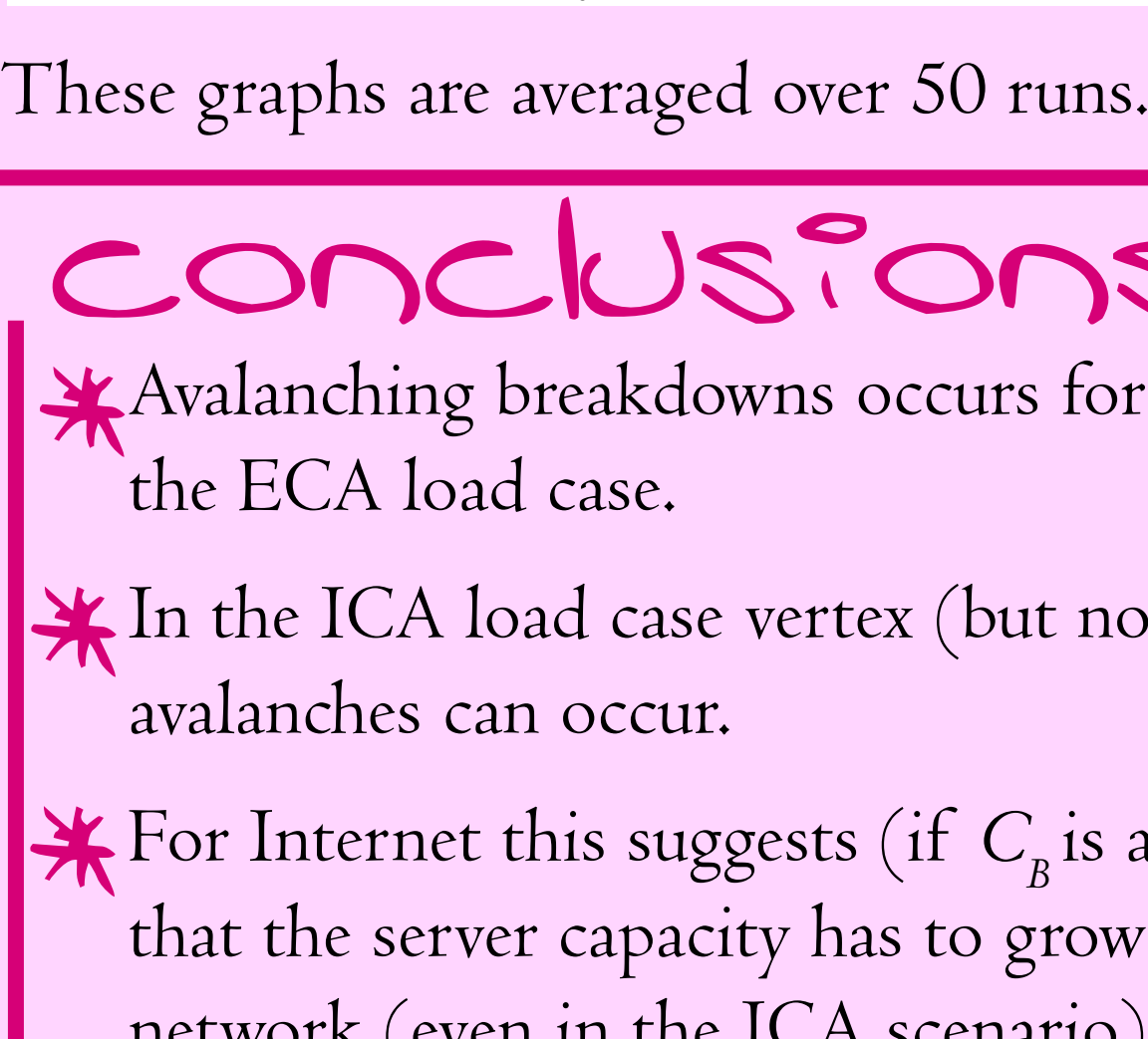
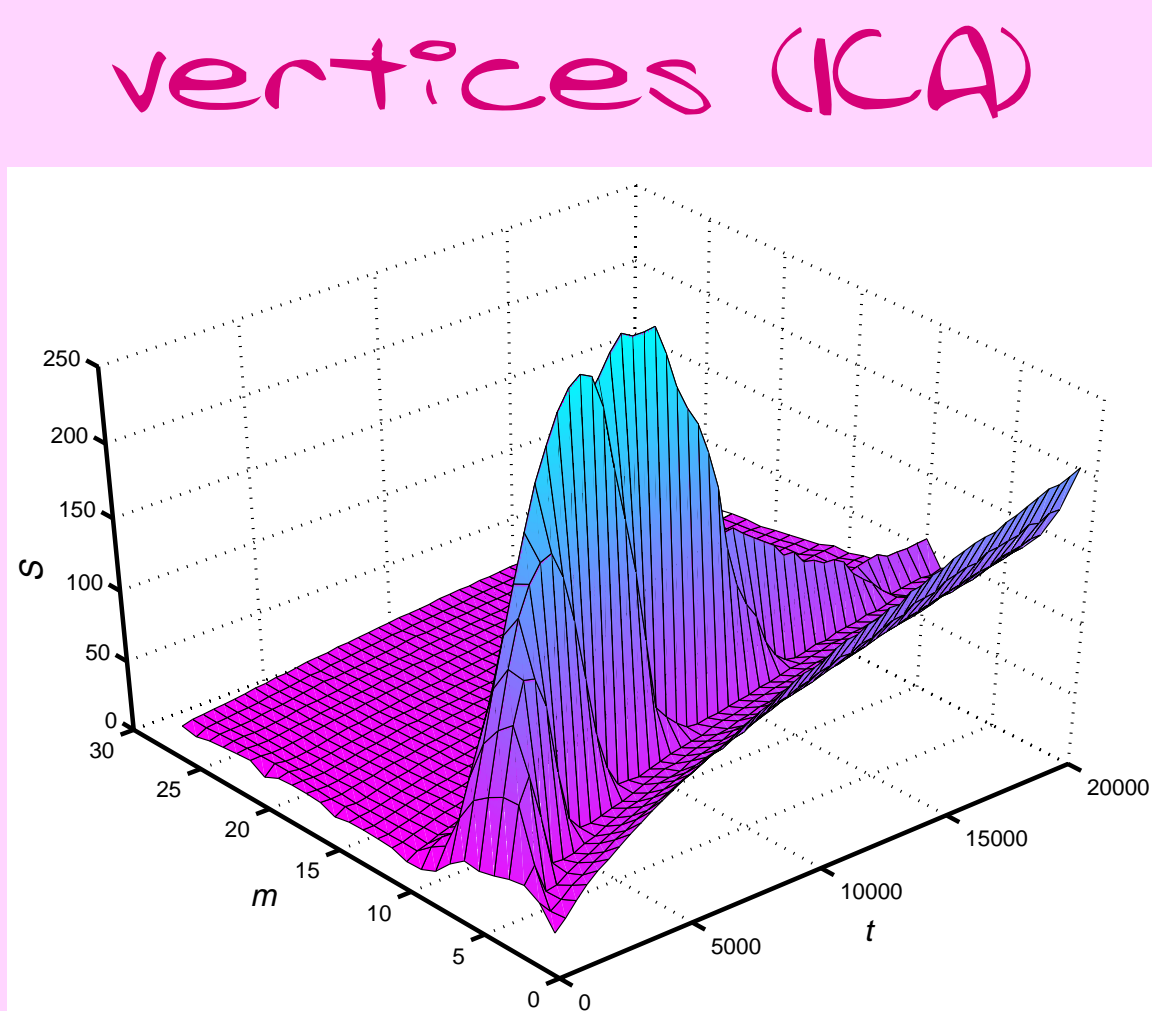
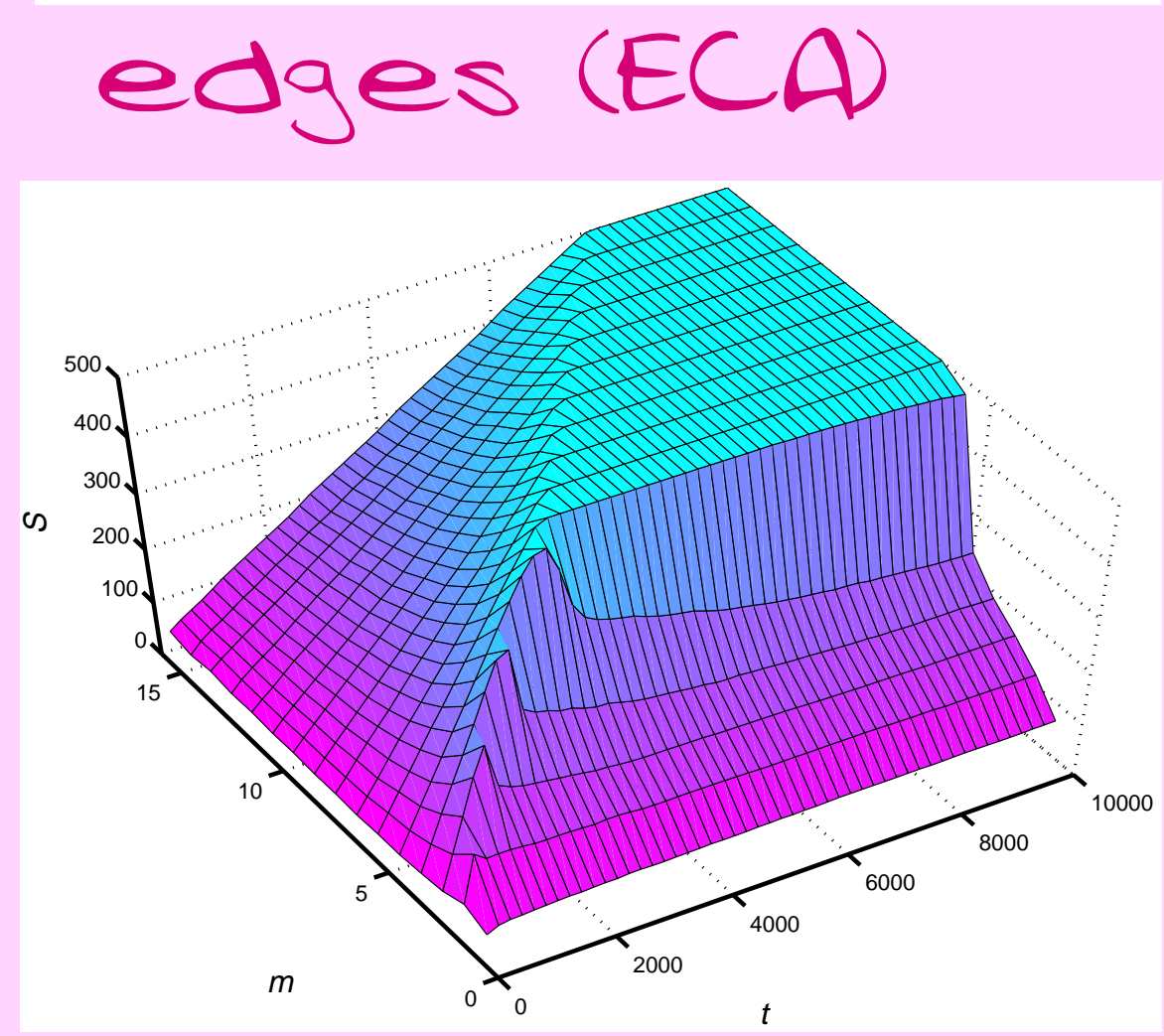
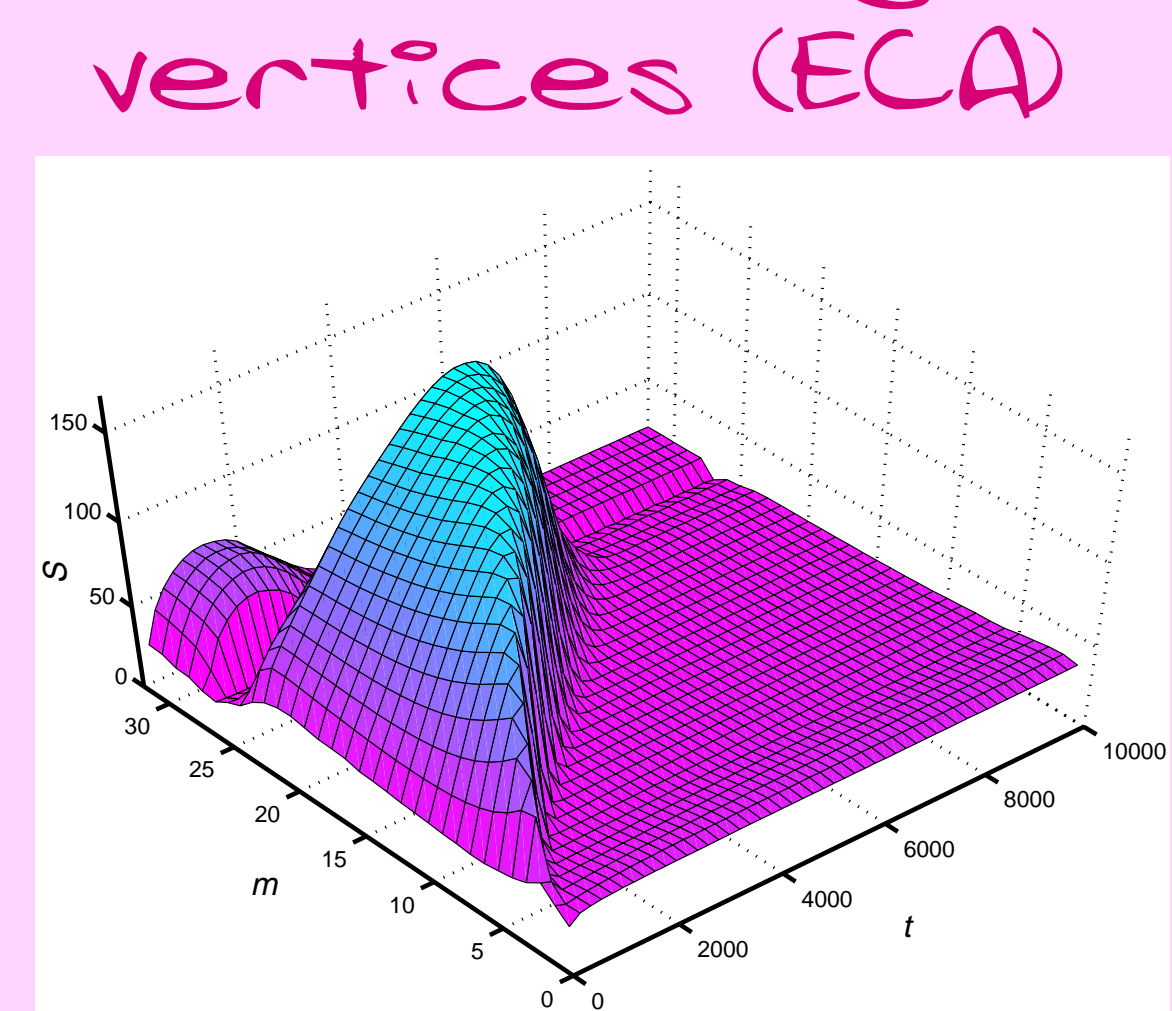
## motivation

- \* Avalanching breakdowns do occur in (communication power distribution etc.) networks.
- \* Little work has been done on overload avalanches and cascading failures.
- \* The redistribution and increase of load in a growing network might trigger overload avalanches.

## different load cases

- We distinguish between different load cases:
1. Extrinsic source activity (ECA) corresponding to that the average number of connections per vertex is increasing with the network's size.
  2. Intrinsic source activity (ICA) corresponding to a constant average number of connections per vertex.
- Real communication networks can be supposed to lie between the ECA and ICA extremes.

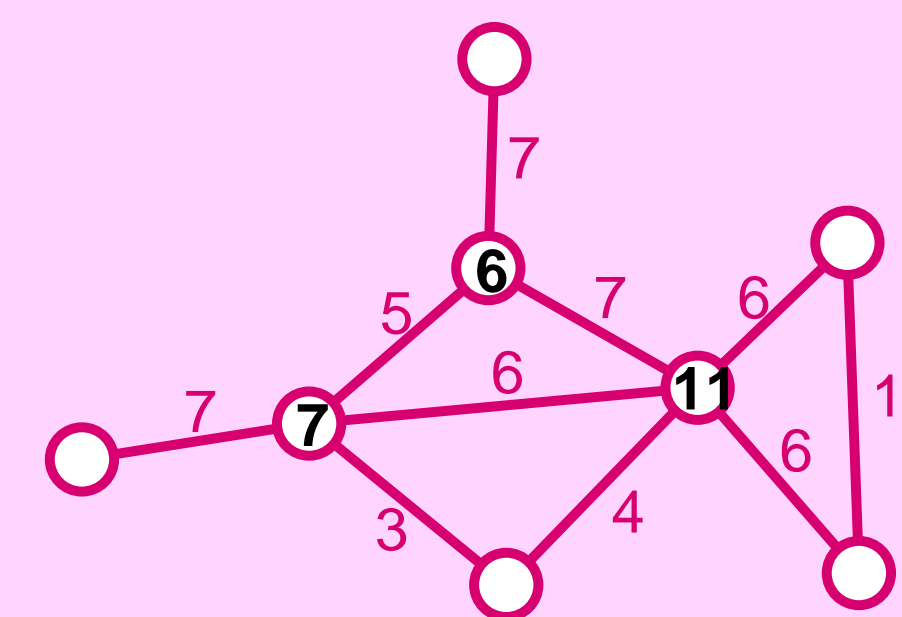
## m scaling of S:



These graphs are averaged over 50 runs.

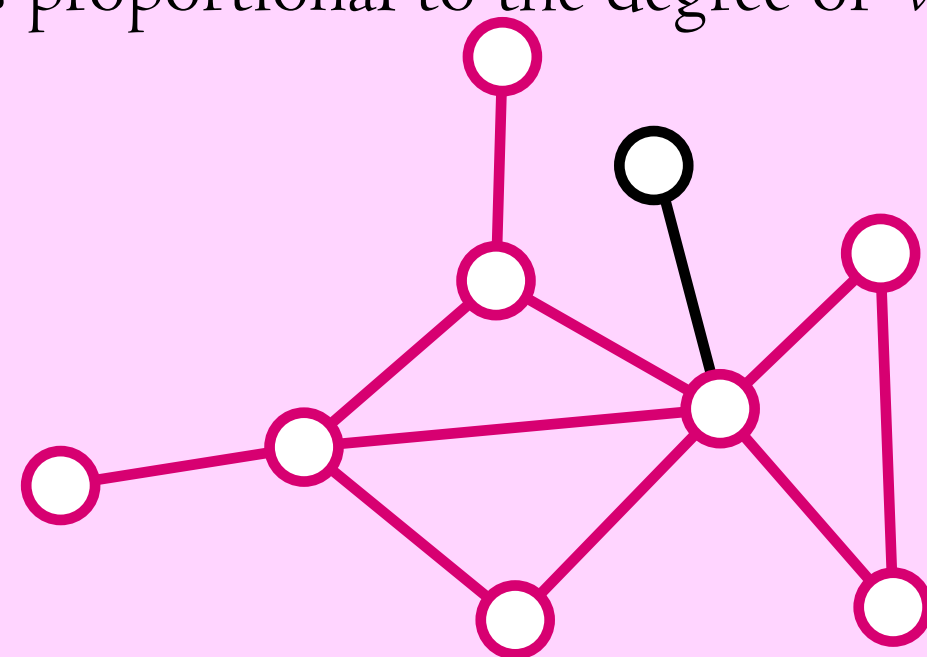
## the load — the growth

We use betweenness centrality  $C_B$  as principle load measure for both vertices and edges.  $C_B$  is count of the geodesics passing a specific vertex or edge:



The network is grown according to the Barabási-Albert model:

1. Initial condition: Start with  $m_0$  vertices.
2. Growth: Add one vertex and  $m$  edges at each iteration.
3. Preferential attachment: The probability of a new vertex to attach to a specific vertex  $v$  is proportional to the degree of  $v$ .



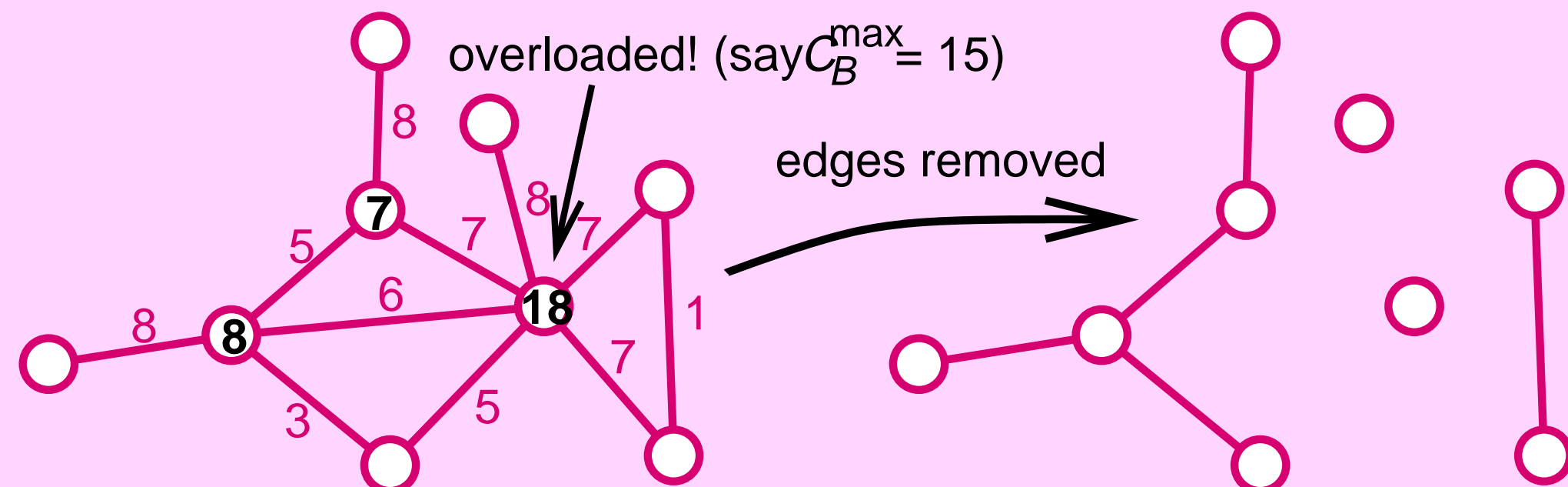
## the breakdowns

### vertices:

If the betweenness of one vertex exceeds a fixed  $C_B^{\max}$  all adjacent edges are removed and the betweenness recalculated.

### edges:

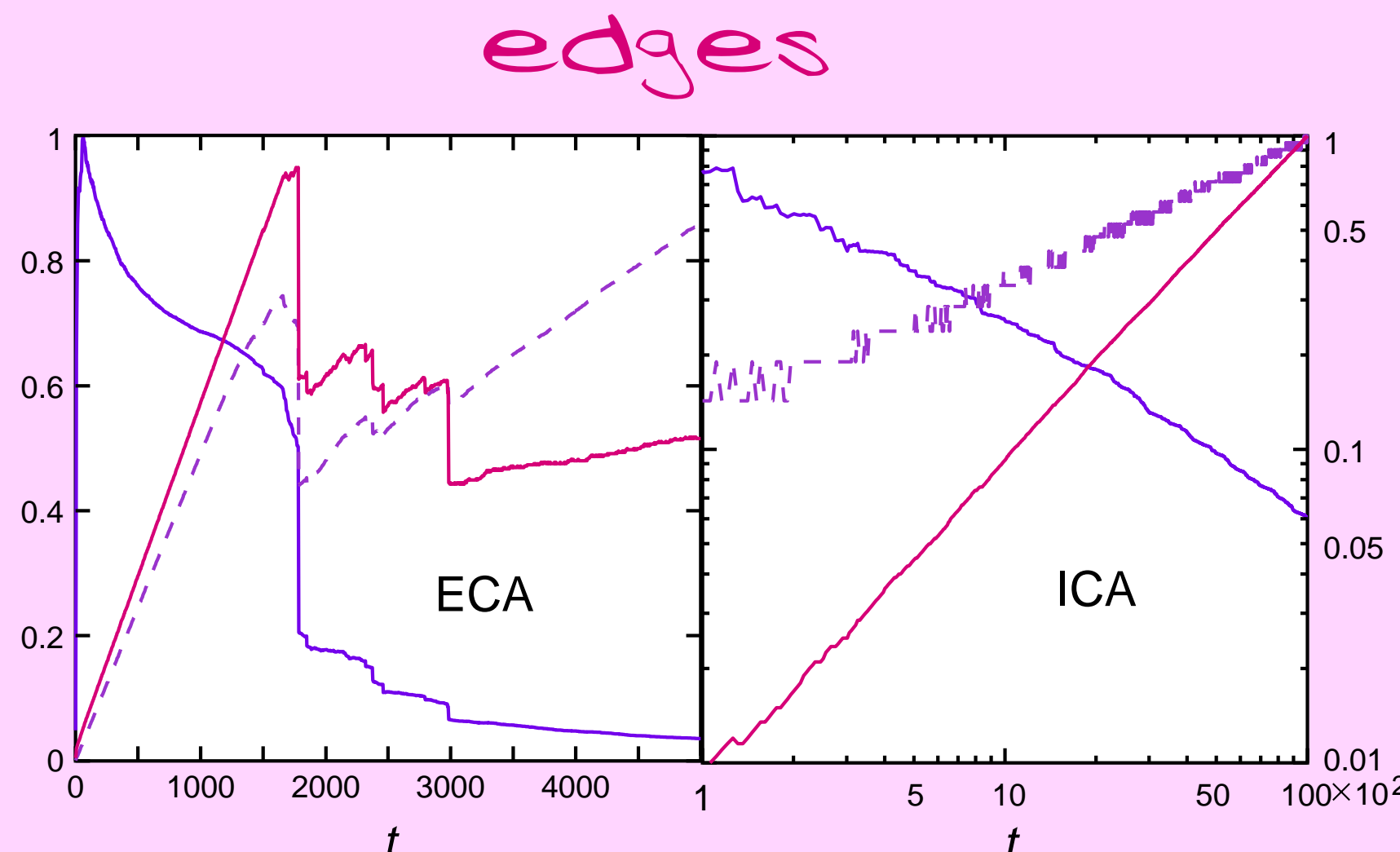
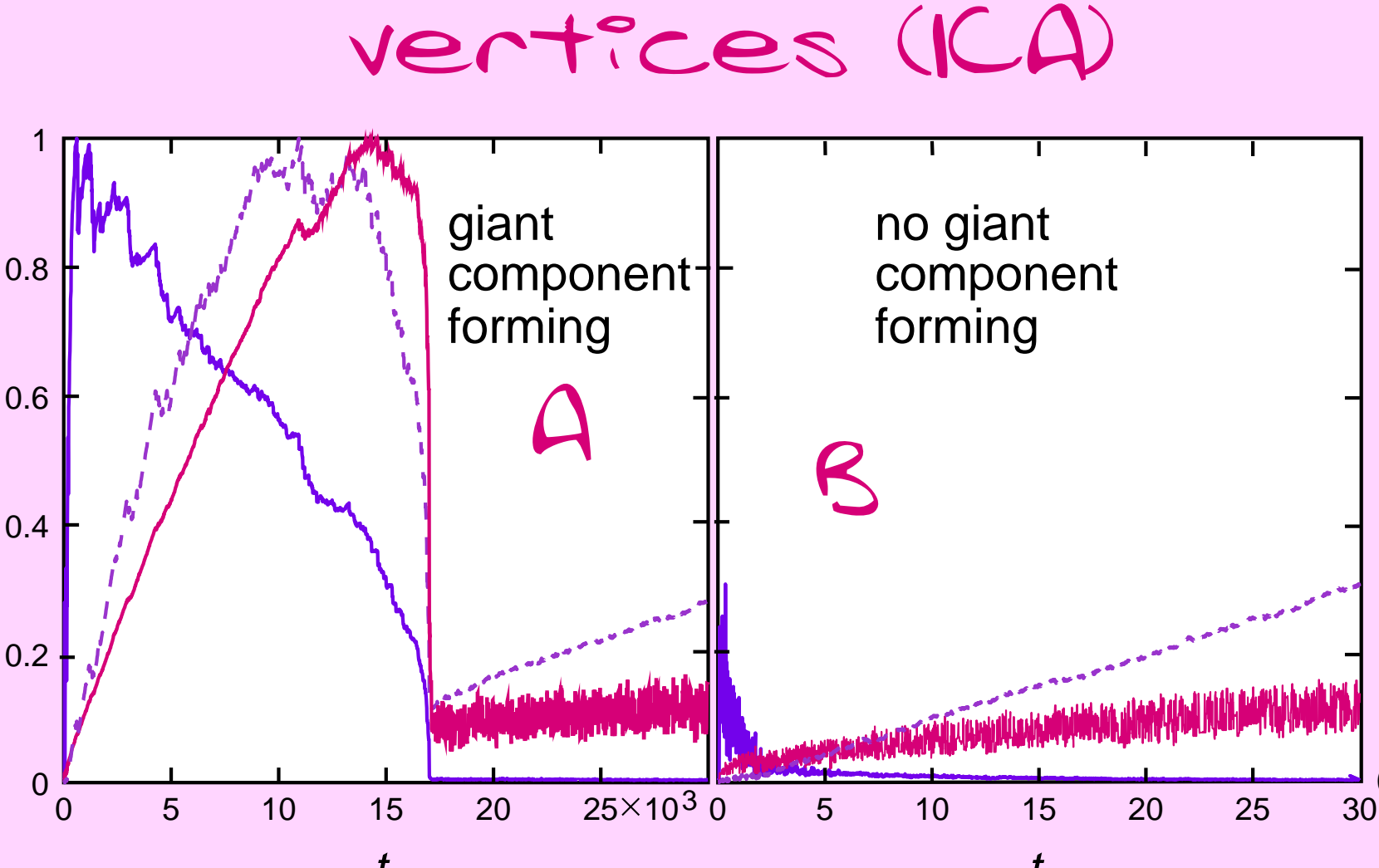
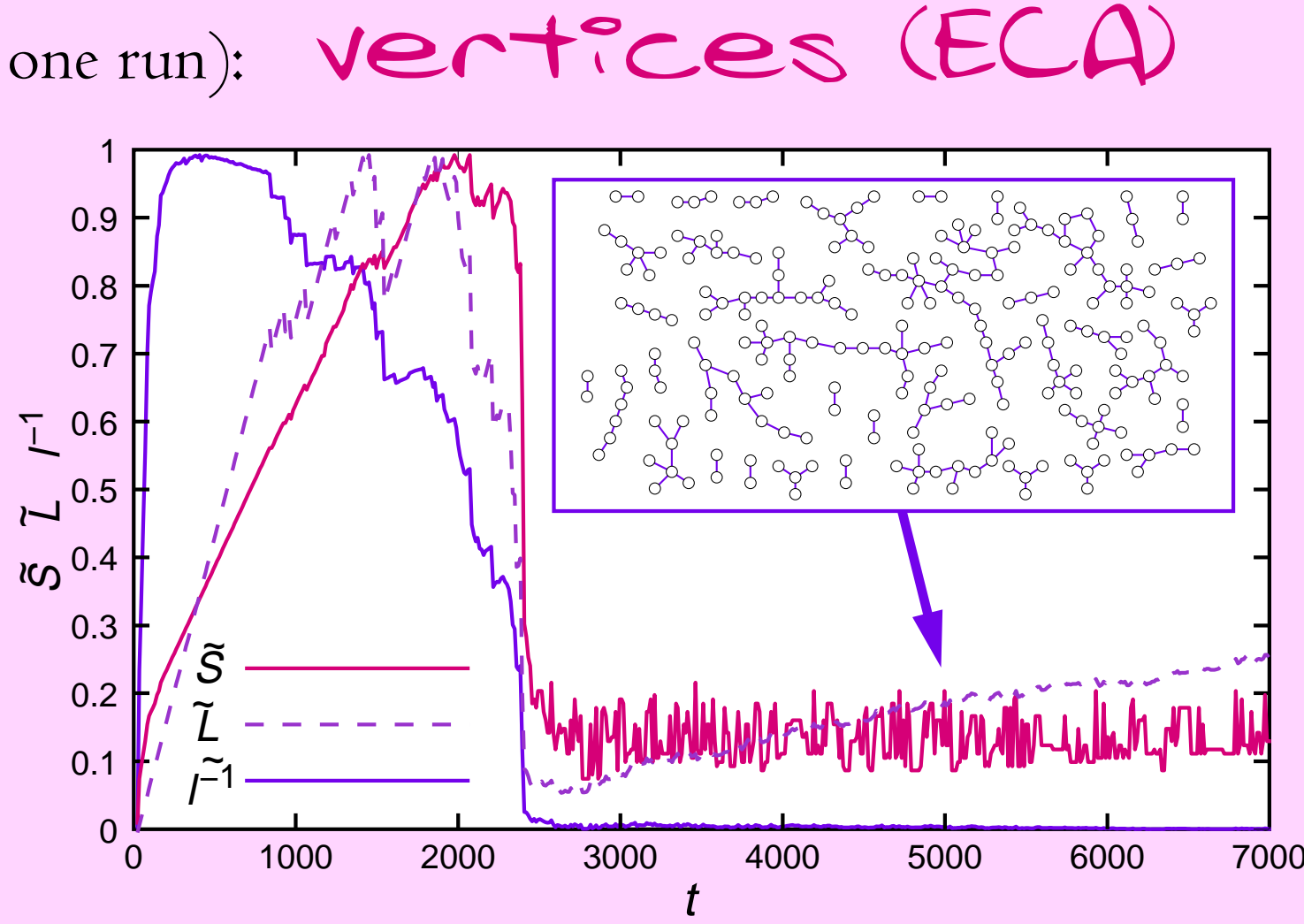
If the betweenness of one edge exceeds a fixed  $C_B^{\max}$  that edge is removed and the betweenness recalculated.



## time evolution:

Here we measure (for one run):

- \* The size of the largest cluster  $S$ .
- \* The average inverse geodesic length  $l^{-1}$ .
- \* The number of edges  $L$ .

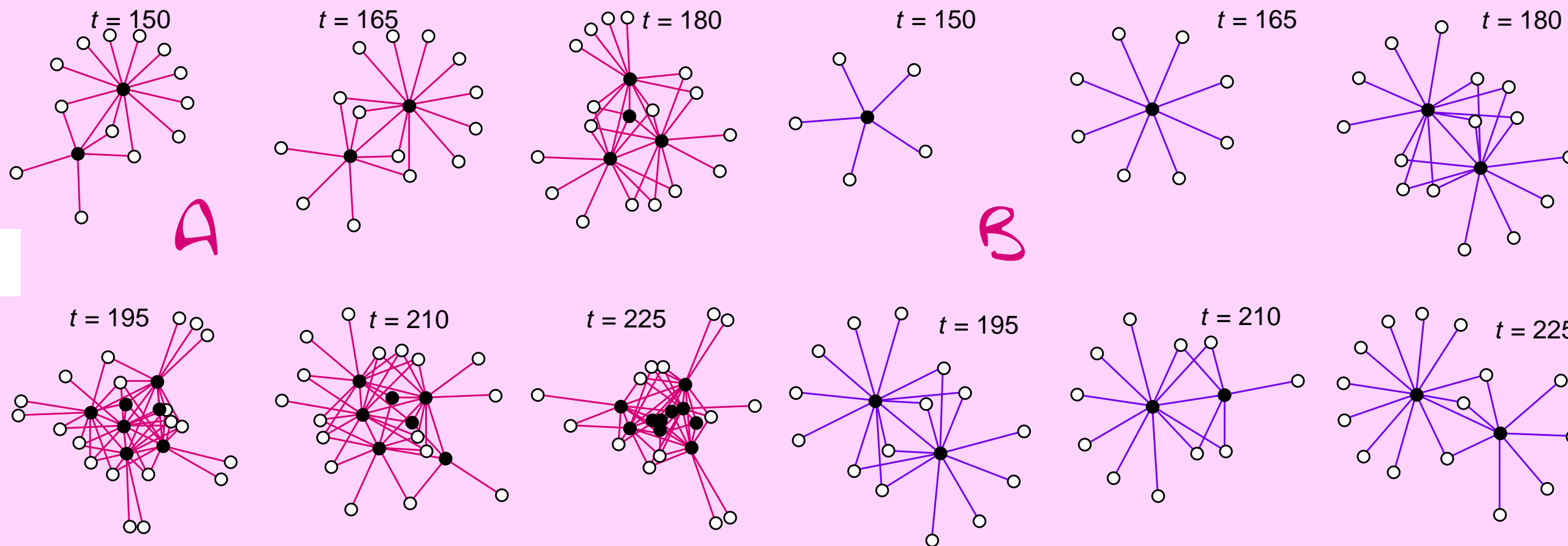
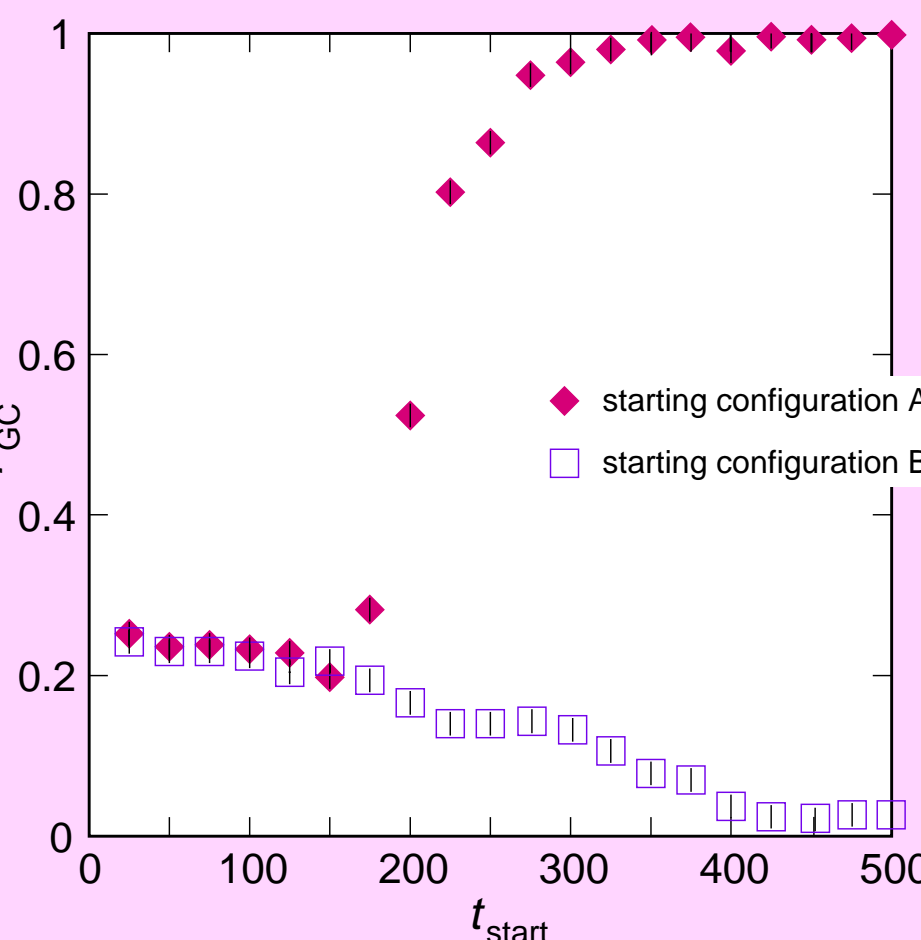


## irregular dynamics:

In the ICA load case for vertices (for some parameter values): A giant component forms only a fraction of the runs.

Here we test the probability for a giant component to form, starting at time  $t$  with the network as in run A (where a g.c. forms) and run B (where no g.c. forms).

(Black vertices has nonzero betweenness.)

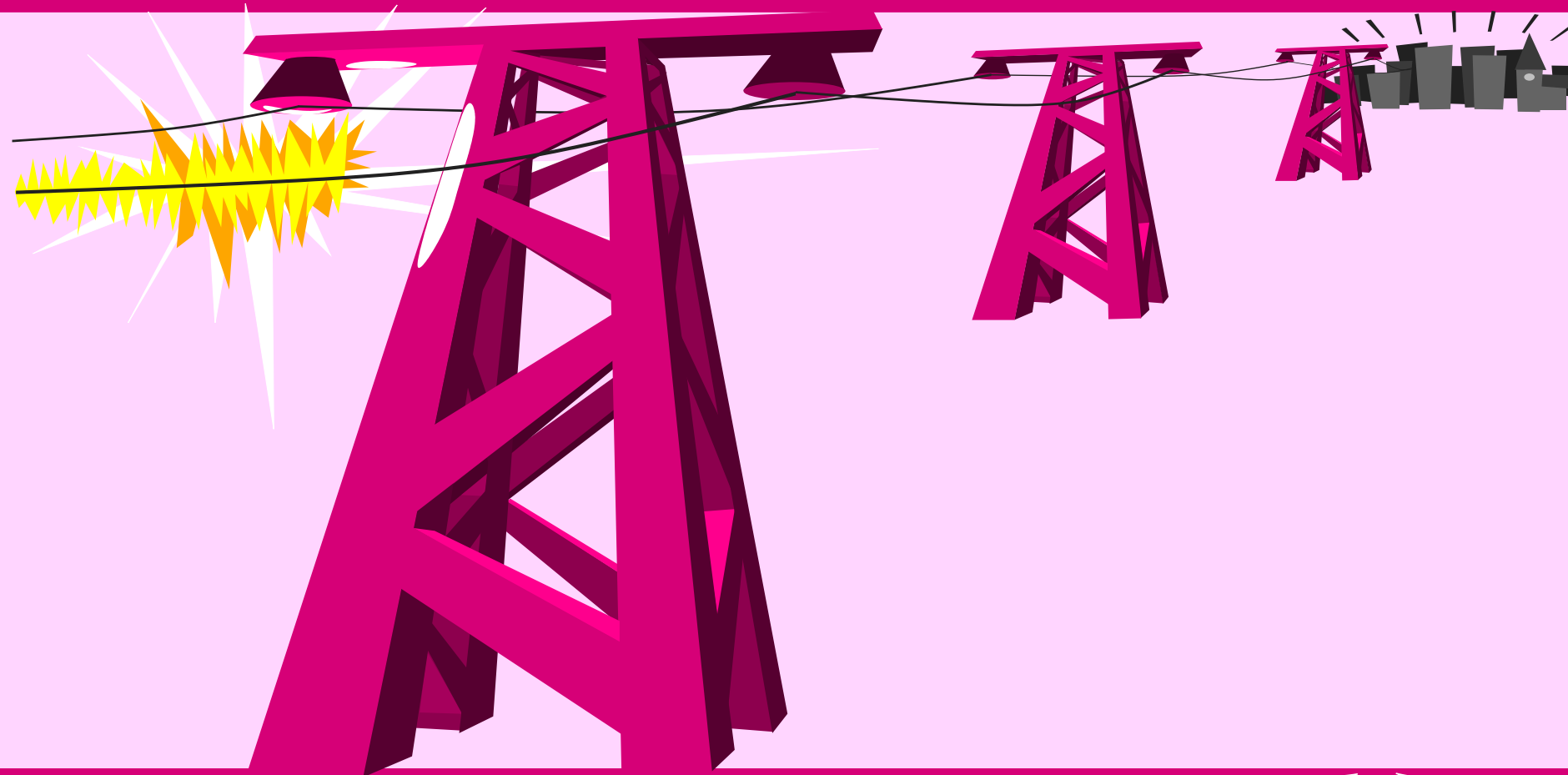


## conclusions:

- \* Avalanching breakdowns occurs for both vertices and edges in the ECA load case.
- \* In the ICA load case vertex (but not edge) breakdown avalanches can occur.
- \* For Internet this suggests (if  $C_B$  is a relevant load measure) that the server capacity has to grow with the size of the network (even in the ICA scenario).
- \* For vertices, the networks are most robust at intermediate  $m$ .
- \* Interesting dynamics of giant component formation in the ICA case for vertices.

## the future:

- \* More realistic static load measures (taking load balancing into account).
- \* Load from dynamical simulations of network flow.
- \* Analytical studies (tough since the network structure changes during a breakdown avalanche).



based on: Petter Holme and Beom Jun Kim *Vertex overload breakdown in evolving networks*, to be published in Phys. Rev. E, e-print cond-mat/0204120.

Petter Holme *Edge overload breakdown in evolving networks*, submitted to Phys. Rev. E.

contact info:

Petter Holme  
Department of Theoretical Physics  
Umeå University  
901 87 Umeå, Sweden

email: holme@tp.umu.se  
homepage: www.tp.umu.se/~holme/  
tel.: +46-90-786 7760  
fax.: +46-90-786 9556

Beom Jun Kim  
Department of Molecular Science and Technology  
Ajou University  
Suwon 442-749, Korea

email: beomjun@ajou.ac.kr  
homepage: www.tp.umu.se/~kim/  
tel.: +82-31-219 2571  
fax.: +82-31-219 1615

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